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SOME NOTES ON COMPOSITE SECOND AND THIRD ORDER INTERMODULATION DISTORTIONS

COMPOSITE THIRD ORDER INTERMODULATION DISTORTION

Composite Third Order distortion is also known as Composite Triple Beat (CTB), and by a few other unmentionable names. Third order distortion products are the result of one, two, three or more carriers experiencing a third order non-linearity.

The NCTA RECOMMENDED PRACTICES FOR MEASUREMENTS ON CABLE TELEVISION SYSTEMS defines CTB as "A composite third order distortion of the desired signals caused by third order curvature of non-linear transfer characteristic in system equipment. It is the ratio, expressed in dB, of the peak of the RF signal to the average level of the cluster of distortion components centered around the carrier."

Fig. 2 on page 5 displays all the distortion products on one channel of a CATV system. In this example there are 8 different products generated.

The strongest and most important third order products are the result of three frequencies. These can be expressed as:

$$A + / - B + / - C \qquad \text{where } A < B < C$$

It is valuable to further categorize these products as:

$$A + B + C, \quad A + B - C, \quad A - B + C, \quad A - B - C \qquad \text{where } A < B < C$$

One distortion product generated by the third order non-linearity is **3A**, three times any frequency. There are of course N of these products where N=number of carriers. These products are 15.6 dB weaker than the **ABC** products. These products do not fall near a carrier and usually are ignored.

A second third order distortion product that falls near the carrier is the **2A - B** product where **A** $\neq$ **B**. This product is one half $(-6\ dB)$ of the magnitude of the ABC product and they are fewer in number. In a system with 20 channels the contribution of the **2A - B** distortion product is less then 0.1 dB and its contribution decreases with increasing number of channels.

There is a third product generated that is a result of the offset channels 4 and 5. These generate products that are 750 KHz offset from the carriers. These products are the same level as the ABC products, but are few in number and are usually ignored.

A fourth product is the **A + B + C** term. This product is also the same level as the other ABC products but also does not fall near a carrier but rather 500 KHz above a carrier. Most of these products fall above the band. These products are also usually ignored.

We now arrive at the most important third order products. These distortion products are important because they fall near the carriers, they are the strongest products and there are many more of them. The expressions for the important third order beats becomes:

$$\boxed{\textbf{A + B - C, \ A - B + C and A - B - C \quad where A < B < C}}$$

With our choice of terms we can have negative frequencies which are just as real as positive frequencies. The reason for our choice is that unless we are careful we can count some beats twice. If the beat products are positive or negative there is no problem but in some cases, **A + B - C** for example, the beats go from positive to negative frequency. Here the positive frequencies fall on channels but the reflected negative frequencies do not. Extra care must be used to avoid errors in evaluating the number of beats.

For the case of equally spaced carriers, a simple expression can be used to estimate the number of third order beats on any channel.

Number of beats = ((N - 1)^2/4 + (N - M)(M - 1)/2) - (N/4)

where **N** = total number of channels
and   **M** = number of the channel being measured

For **N >> 1**, we may use **N – 1 = N** and **N/4 = 0**

Number of beats = **N^2 /4 + (N - M)(M - 1)/2**

In the middle of the band this expression simplifies to:

Number of beats (mid band) **= 3 N ^{2}/ 8**

At the edges of the band the expression simplifies to:

Number of beats (band edge) **= N ^{2}/ 4**

It is interesting to note that the number of beats at the band edges is 2/3 (-1.76 dB) of the number of beats in the middle of the band. This ratio is independent of the number of channels. This expression was derived by Dr. Thomas B. Warren and has been shown to be accurate to within 1%.

The above analysis is for a normal system with carriers that are not phase locked. In systems with carriers that are not phase locked, the beats add as powers because they are all at different frequencies.

 In a phase locked (HRC) system the beats on any one channel are at exactly the same frequency as the carrier and in the worst case would add as voltages. Their phases are usually random so these beats are usually considered as adding as powers. Under operating conditions the beats of an HRC system are invisible because they fall directly under the carrier.

ENVELOPE OF SECOND AND THIRD ORDER DISTORTION PRODUCTS VS FREQUENCY
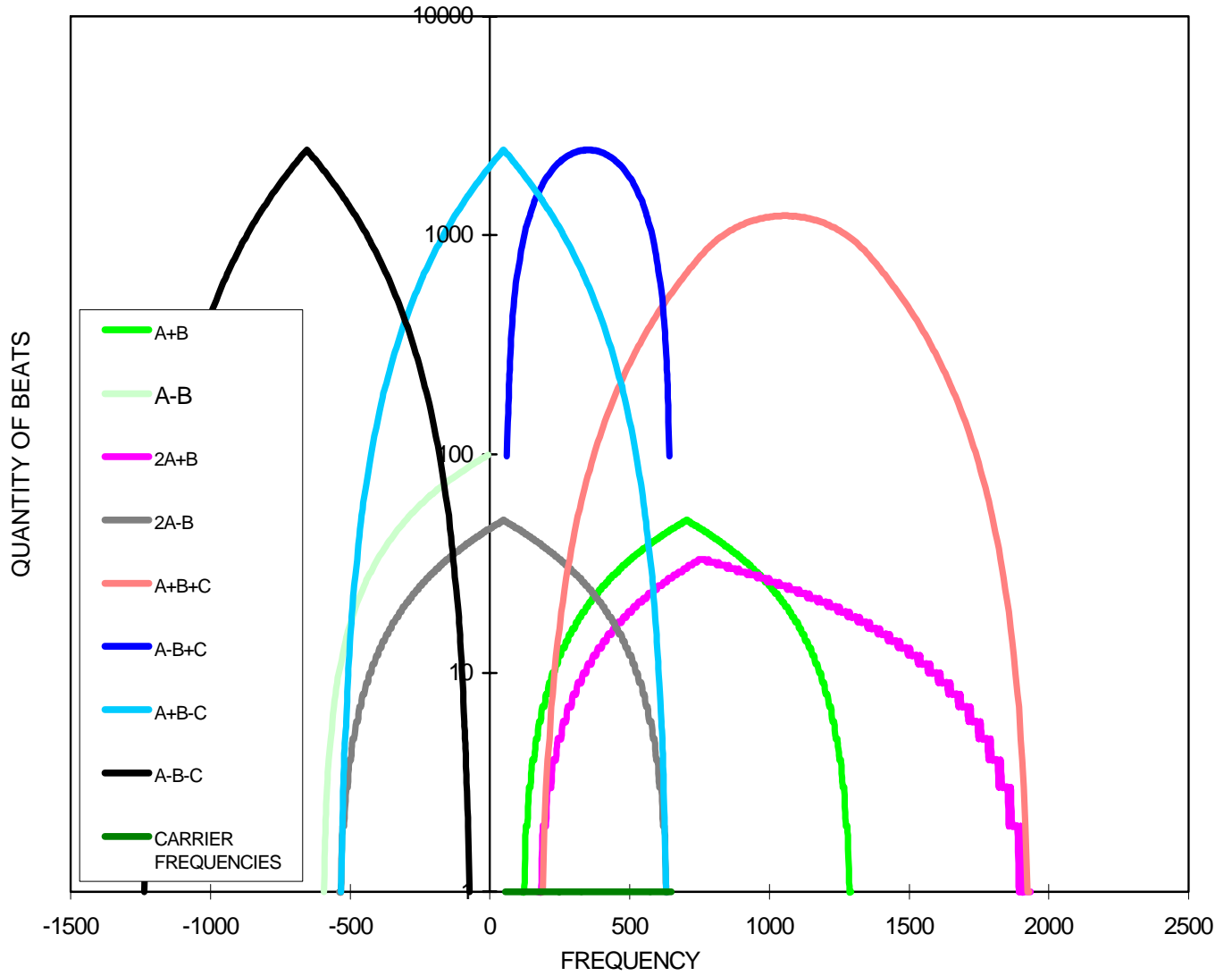## FOR 100 EQUALLY SPACED CARRIERS

Legend:
- A+B
- A-B
- 2A+B
- 2A-B
- A+B+C
- A-B+C
- A+B-C
- A-B-C
- CARRIER FREQUENCIES

Y-axis: QUANTITY OF BEATS (10000, 1000, 100, 10, 1)

X-axis: FREQUENCY (-1500, -1000, -500, 0, 500, 1000, 1500, 2000, 2500)
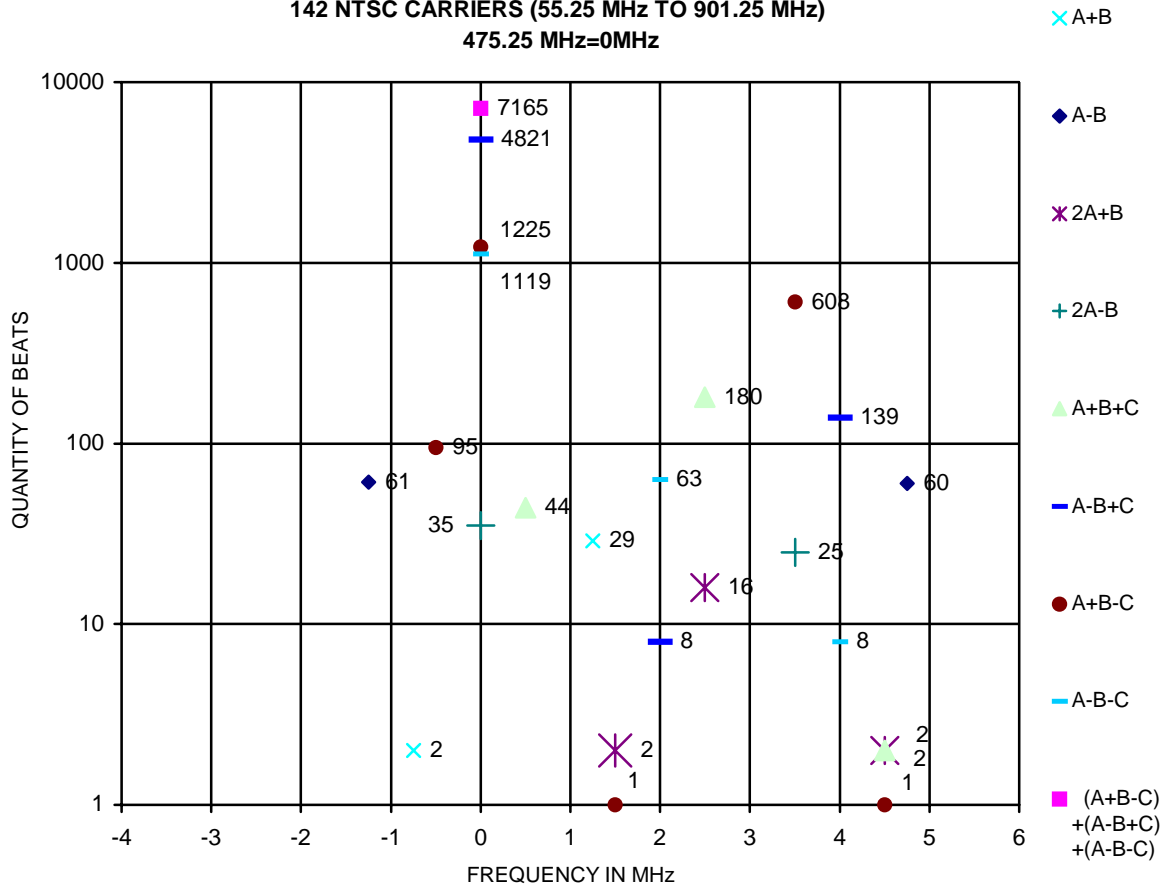
FIG 1

FIG 2

**SECOND ORDER AND SOME THIRD ORDER DISTORTION PRODUCTS VS FREQUENCY
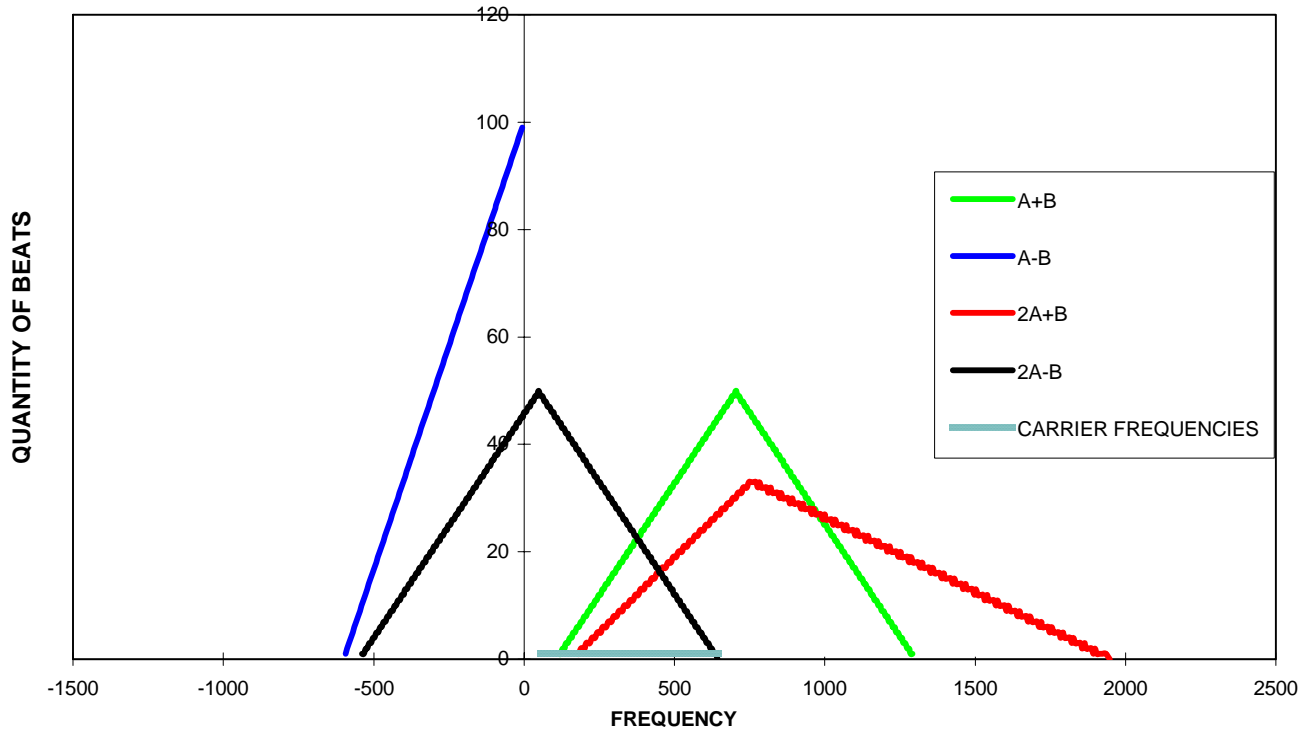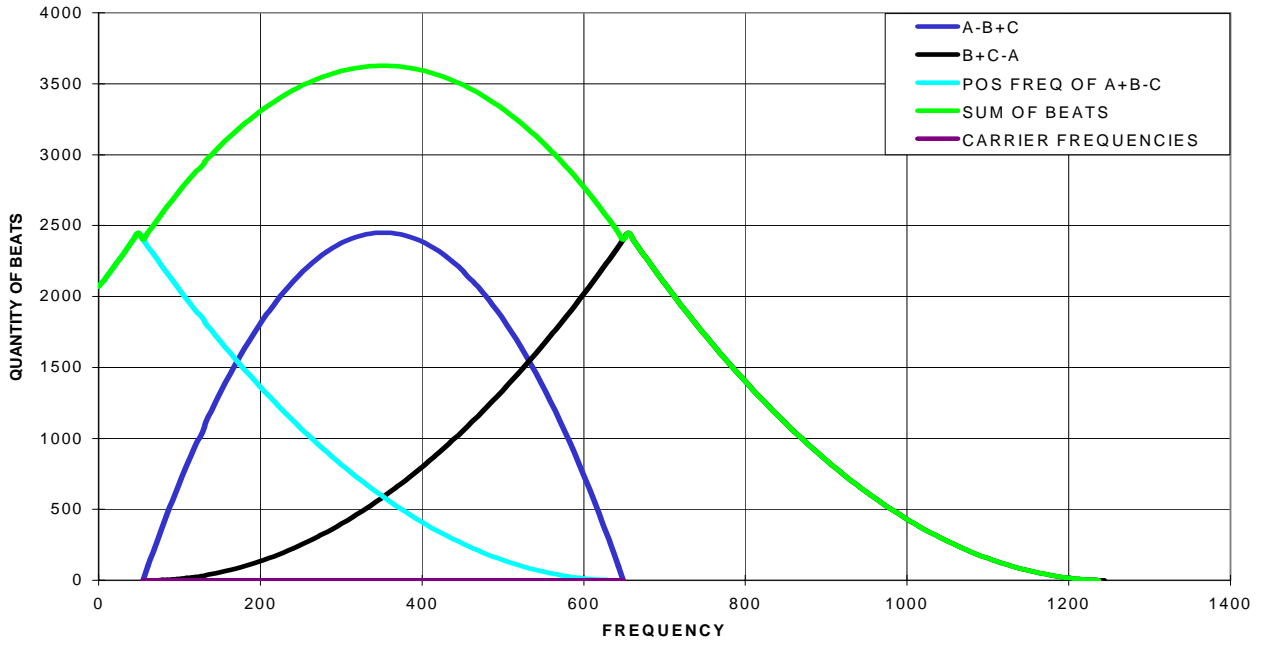100 EQUALLY SPACED CARRIERS**

FIG 3

FIG 1 is a graph showing the envelope of the number of beats for 100 carriers. FIG 3 is a linear representation of some of the second and third order distortions. The reason for this graph is to show that these curves are actually straight lines, a fact that is hidden in FIG 1.

FIG 2 shows the quantity of second and third order products and their frequency in one particular video channel. The quantity of these products varies with the particular channel with their envelopes shown in FIG 1. For the example given here the standard NTSC frequency allocation was chosen rather than equally space carriers. This of course results in a better model of what actually exists.
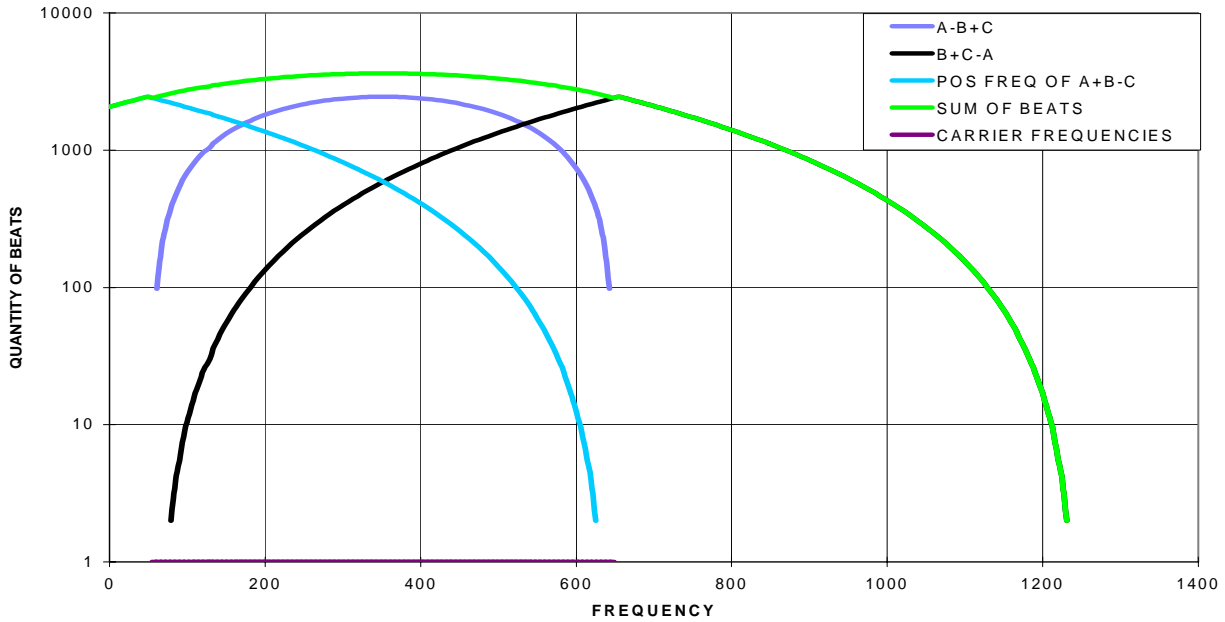
The main third order products are shown in FIG 4 and FIG 5. Also shown is the sum envelope that is in close agreement with the expression for the total number of beats shown above.

DOMINANT THIRD ORDER BEATS VS FREQUENCY FOR 100 EQUALLY SPACED CARRIERS

FIG 4

DOMINANT THIRD ORDER BEATS VS FREQUENCY FOR 100 EQUALLY SPACED CARRIERS

FIG 5

COMPOSITE SECOND ORDER INTERMODULATION DISTORTION

Composite Second Order distortion (CSO) is a result of one or two carriers experiencing a second order non-linearity. An argument similar to the third order distortion can be made here.

One second order product is the **2A** term. This product is at two times each carrier frequency and therefore there is exactly one beat for each carrier frequency. The products fall 250 KHz above the carriers with the lowest frequency beat being at two times the lowest carrier frequency. This beat is one half (-6dB) of the magnitude of the **A + B** and the **A - B** beats and is usually ignored.

The important second order beats are the **A + B** and the **A - B** terms with **A < B**. It is true that **A - B** is a negative frequency but this only means that these products fold around zero frequency and do actually fall in band. See FIG 6 for a graphic representation of these products.
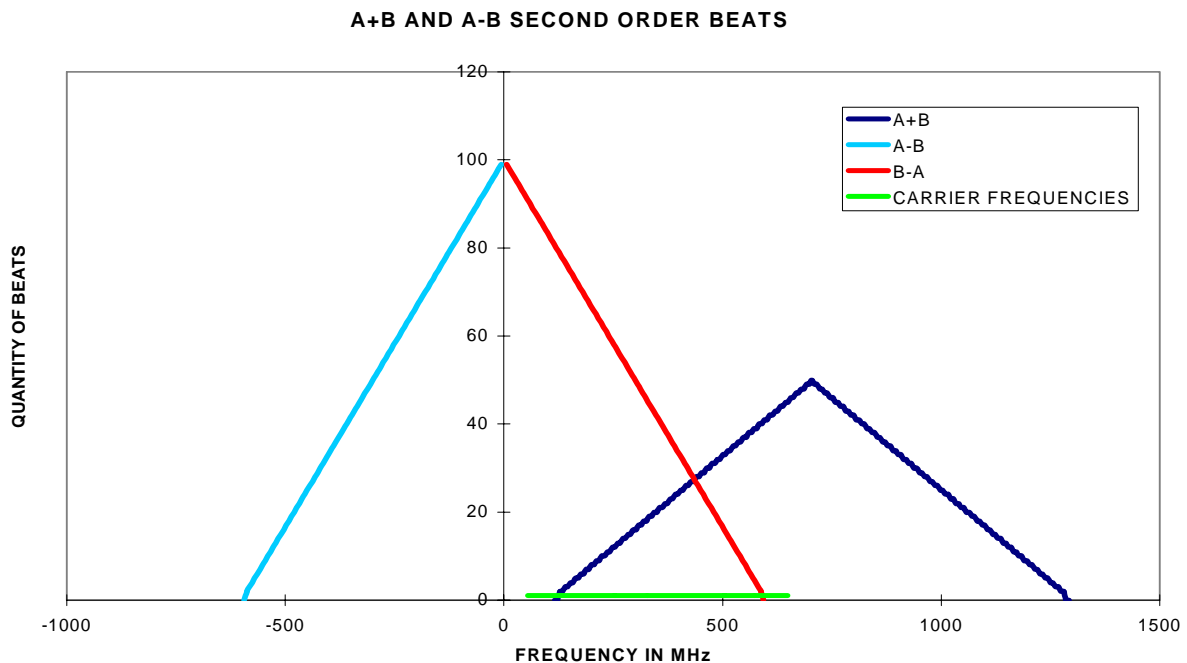

FIG 6

The **A + B** terms start at a frequency that is the sum of the first and second carrier frequencies and end at a frequency that is the sum of the highest and next to highest frequency. The **A + B** products fall 1.25 MHz above the carrier of the channel. It is also clear that not all these beats fall in band. The maximum number of **A + B** beats is N/2, where N=number of channels. The frequency of this maximum is at the sum of the highest and lowest carrier frequencies. This is obviously above the highest frequency channel.

The **A - B** terms fall 1.25 MHz below the carrier frequencies. The lowest frequency **A - B** products are at 6 MHz, the carrier spacing. This is the point of maximum number of beats which is N-1, where N=number of carriers. The highest frequency **A - B** product is at a frequency which is the (highest frequency) - (lowest frequency). At this frequency only one beat is present.

As in the CTB case when the carriers are HRC or IRC, and phase locked, the beats fall exactly on the carriers.

There are two useful expressions that can be used to find the number of second order beats on any channel. These expressions are similar to the one for CTB are not exact, but the errors are very small. These expressions were also derived by Dr. Thomas B. Warren.

To calculate the number of second order beats that fall below the carrier use the following formula. Equal carrier spacing is assumed. Frequencies are in MHz.

Number of beats (Below carrier) $= (N - 1)(1 - ((f - d) / (f_H - f_L)))$
     for $0 < f < (f_H - f_L)$

    N = Number of carriers
    f = Frequency of measurement channel
    f_H = Frequency of highest frequency channel
    f_L = Frequency of lowest frequency channel
    d = Frequency offset from a multiple of 6 MHz. (1.25 MHz.)

    N is also equal to $((f_H - f_L) / 6) + 1$

To calculate the number of second order beats that fall above the carrier use the following.

Number of beats (Above carrier frequency) $= (N - 1)((f - 2f_L + d) / 2 (f_H - f_L))$

For $2 f_L < f < (f_H + f_L)$ and terms are as defined above.

RELATIVE MAGNITUDE OF SECOND AND THIRD ORDER DISTORTION PRODUCTS

In all of the above discussions, only the quantity of beats has been addressed. The relative strength of the distortion products is also an important consideration. Clearly if the second order products are 30 dB stronger that the third order products, the second order products may dominate although there are fewer of then. Matrix Test Equipment Inc. Application note MTN-109 presents a discussion covering the relative strength of the various second and third order distortion products.

Of prime importance, for the examples presented here is the relative strength of the second order and third order distortion products. If we consider a system with three frequencies, **A**, **B** and **C**, the strongest distortion products are:

A +/- B   for second order

A + B + C, A + B - C, A - B + C, A - B - C   for third order

In a well-behaved amplifier with a frequency response wide enough to cover all the distortion frequencies, the level of second order products will be of equal level. The third order products will also be of equal level. The ratio of the second order products to the third order products greatly depends upon the type of device generating the distortion. For common single ended devices, operating within their design limits, the magnitude of the individual second order products will be much greater than the magnitude of the individual third order products. Push pull operation, or some other balanced configuration, allows reduction of the second order products but not the third order products.

POTENTIAL SOURCES OF ERRORS

The above measurements are made using a spectrum analyzer in the LOG mode. This may seem to be an obvious statement, but because spectrum analyzers read noise and noise-like signals in error, other methods of making these measurements cause discrepancies. If the ratio of the peak of the carrier to the power in the distortion products were to be measured using a power meter, or some other device that reads true power, a discrepancy of about 2.5 dB would be seen. For the example given, the spectrum analyzer reads the noise-like signals as being weaker and, therefore, displays a higher (better) ratio.

One further associated problem is related to the crest factor of the distortion. The crest factor is the peak to average ratio of the distortion. A time measurement of the composite distortion shows a predominant signal with a frequency equal to the carrier spacing. The peak to average of the waveform is dependent to some extent upon the accuracy of the carrier frequencies. The detector in the spectrum analyzer does not respond well to signals with large crest factors. Errors of several dB have been seen with variations in the frequency spread in a normal system. Tests done with coherent carriers with crest factors of 10 dB show spectrum analyzer errors of over 10 dB. These measurements are greatly dependent upon the choice of video bandwidth.

Fortunately the method of making these measurements shown in MATRIX **MTN-107** and **MTN-110** are not affected by the crest factors normally encountered in these measurements. Actual measurements on coherent signals adjusted for maximum and minimum crest factors show no difference in the reading.

All of the formulas used were derived by
Dr. Thomas B. Warren, Matrix Test Equipment inc.
Interpretations, opinions, explanations and other errors are the responsibility of
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